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EFFECT OF TEMPERATURE ON SLIP COEFFICIENTS OF MOLECULAR GASES

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Results obtained by accurate analytical methods applied to the problem of molecular-gas slip over a rigid spherical surface are reported. The Boltzmann equation is modified to take into account rotational degrees of freedom in the BGK *model is used as a master kinetic equation. The calculated slip coefficients are shown to depend on the Prandtl number and on the gas temperature. Slip coefficients for several molecular gases are plotted as functions of temperature.*

Key words: *rarefied gas, slip coefficient, aerosol particles.*

Introduction. Result reported up to now and related to imposing boundary conditions on surfaces exposed to a rarefied gas primarily refer to simple monatomic gases. Exceptions here are the studies [1–6], where boundary-value problems for polyatomic gases were treated and it was shown that allowance for the internal structure of gas molecules leads to substantial corrections to quantities reflecting the gas–surface interaction.

The objective of the present study was to calculate the slip coefficients for a molecular-gas flow over the surface of a spherical aerosol particle with a small radius of curvature ($\mathrm{Kn} = \lambda/R' \ll 0.02$, where Kn is the Knudsen number, R' is the radius of the aerosol particle, and λ is the mean free path of gas molecules).

The master kinetic equation was assumed to be the Boltzmann equation modified to take into account rotational degrees of freedom of gas molecules in the Bhatnagar–Gross–Krook (BGK) model [1]:

$$C_r \frac{\partial Y}{\partial r} + Y(\boldsymbol{r}, \boldsymbol{C}, \nu) + k\Big[C_\theta \frac{\partial Y}{\partial \theta} + \frac{C_\varphi}{\sin\theta}\frac{\partial Y}{\partial \varphi} + (C_\theta^2 + C_\varphi^2)\frac{\partial Y}{\partial C_r} + (C_\varphi^2 \cot\theta - C_r C_\theta)\frac{\partial Y}{\partial C_\theta}$$

$$- (C_\varphi C_\theta \cot\theta + C_r C_\varphi)\frac{\partial Y}{\partial C_\varphi}\Big] = \int K(\boldsymbol{C}, \nu, \boldsymbol{C}', \nu') Y(\boldsymbol{r}, \boldsymbol{C}', \nu')\, d\Omega.$$

Here $l = 2$, $d\Omega = 2\pi^{-3/2} \exp(-C^2 - \nu^2)\nu \, d\nu \, d^3C$ for a diatomic gas, $l = 5/2$ and $d\Omega = \pi^{-3}\exp(-C^2 - \nu^2)\, d\nu \, d^3C$ for a polyatomic gas (with the number of atoms in a molecule $N \geqslant 3$),

$$K(\boldsymbol{C}, \nu, \boldsymbol{C}', \nu') = 1 + 2\boldsymbol{C}\boldsymbol{C}' + (C^2 + \nu^2 - l - 1/2)(C'^2 + \nu'^2 - l - 1/2)/(l + 1/2),$$

$k = 4\mathrm{Kn}/(3\sqrt{\pi}\,\mathrm{Pr})$, and $\boldsymbol{r} = 3\sqrt{\pi}\,\mathrm{Pr}/(4\lambda)\boldsymbol{r}'$ in the case of isothermal slip, $k = 2\mathrm{Kn}/(\sqrt{\pi}\,\mathrm{Pr})$ and $\boldsymbol{r} = \sqrt{\pi}\mathrm{Pr}/(2\lambda)\boldsymbol{r}'$ in the case of thermal slip, $\boldsymbol{r}'$ is the dimensional radius-vector, $\lambda = \nu_{\mathrm{g}}\sqrt{\pi m/(2k_{\mathrm{B}}T_w)}$, ν_{g} is the kinematic viscosity of the gas, Pr is the Prandtl number, $\boldsymbol{C} = \boldsymbol{v}\sqrt{m/(2k_{\mathrm{B}}T_w)}$, $\nu = \omega\sqrt{J/(2k_{\mathrm{B}}T_w)}$, $\boldsymbol{v}$ and ω are the translational and rotational velocities of gas molecules, T_w is the particle-surface temperature, k_{B} is the Boltzmann constant, and m and J are the mass of a gas molecule and its moment of inertia.

In deriving this equation, we assumed that rotational degrees of freedom could be treated classically, whereas vibrational degrees of freedom of molecules were "frozen": $|T/T_w - 1| \ll 1$, $\lambda|\nabla \ln T| \ll 1$, and $U'\sqrt{m/(2k_{\mathrm{B}}T_w)} \ll 1$. The equation is written in a spherical coordinate system whose origin coincides with the center of curvature of the

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particle surface; the function $Y(\boldsymbol{r}, \boldsymbol{C}, \nu)$ allows for deviation of the distribution function in the Knudsen layer from the distribution function over the gas volume:

$$f(\boldsymbol{r}', \boldsymbol{v}, \omega) = f^0(\boldsymbol{r}', \boldsymbol{v}, \omega)[1 + Y(\boldsymbol{r}, \boldsymbol{C}, \nu)].$$

We obtain the equation

$$f^0(\boldsymbol{r}', \boldsymbol{v}, \omega) = n\Big(\frac{m}{2\pi k_\mathrm{B}T}\Big)^{3/2} \frac{J}{k_\mathrm{B}T} \exp\Big[-\frac{m(\boldsymbol{v}-\boldsymbol{u})^2}{2k_\mathrm{B}T} - \frac{J\omega^2}{2k_\mathrm{B}T}\Big]$$

for a diatomic gas and

$$f^0(\boldsymbol{r}', \boldsymbol{v}, \omega) = n\Big(\frac{m}{2\pi k_\mathrm{B}T}\Big)^{3/2} \frac{(J_1 J_2 J_3)^{1/2}}{(2\pi k_\mathrm{B}T)^{3/2}} \exp\Big[-\frac{m(\boldsymbol{v}-\boldsymbol{u})^2}{2k_\mathrm{B}T} - \frac{1}{2k_\mathrm{B}T}\sum_{i=1}^{3} J_i\omega_i^2\Big]$$

for a polyatomic gas. Here J_i ($i = 1, 2, 3$) are the components of the moment of inertia of gas molecules. To formulate the boundary condition on the particle surface, we use the model of diffuse reflection. This choice was motivated by the fact that diffusion coefficient is close to unity for most engineering (i. e., not specially prepared) surfaces, which include the surface of aerosol particles. For this reason, the use of more sophisticated models of boundary conditions (such as the specular–diffuse model, for instance) would induce no substantial corrections to slip coefficients but, simultaneously, would have seriously complicated the solution of the problem.

1. Statement of the Problem. Derivation of the Governing Equations. We consider a spherical aerosol particle suspended in a rarefied molecular-gas flow. We attach a spherical coordinate system to the center of curvature of the surface, the polar axis of this coordinate system being directed along the temperature gradient far from the surface.

We assume that a constant temperature gradient ∇T is set far from the surface. Because of the nonuniform distribution of temperature over the gas volume, the derivatives $\partial T/\partial r$ and $\partial T/\partial\theta$ are other than zero on the particle surface. The former derivative results in a temperature jump on the particle surface, and the latter leads to thermal slip of the gas over the surface. We also assume that the normal-to-surface component of the temperature gradient slowly varies over the particle surface. Thus, the problem involves a nonzero mixed derivative $\partial^2 T/\partial r\,\partial\theta$ giving rise to additional slip of the gas over the particle surface (the so-called thermal slip of the second order). Next, we assume that the tangential component of mass velocity in the gas flow slowly varies along the normal to the surface. The nonuniform distribution of mass velocity causes gas slip over the surface, which is called the isothermal slip.

Following [7], we seek for $Y(\boldsymbol{r}, \boldsymbol{C}, \nu)$ as an expansion with respect to the parameter k:

$$Y(\boldsymbol{r}, \boldsymbol{C}, \nu) = kY_1(\boldsymbol{r}, \boldsymbol{C}, \nu) + k^2 Y_2(\boldsymbol{r}, \boldsymbol{C}, \nu) + \dots. \tag{1.1}$$

We substitute (1.1) into the master equation and equate the terms at identical powers of k to obtain the following equations for obtaining the functions $Y_1(\boldsymbol{r}, \boldsymbol{C}, \nu)$ and $Y_2(\boldsymbol{r}, \boldsymbol{C}, \nu)$:

$$C_r\,\frac{\partial Y_1}{\partial r} + Y_1(\boldsymbol{r}, \boldsymbol{C}, \nu) = \int K(\boldsymbol{C}, \nu; \boldsymbol{C}', \nu') Y_1(\boldsymbol{r}, \boldsymbol{C}', \nu')\,d\Omega; \tag{1.2}$$

$$C_r\,\frac{\partial Y_2}{\partial r} + Y_2(\boldsymbol{r}, \boldsymbol{C}, \nu) = \int K(\boldsymbol{C}, \nu; \boldsymbol{C}', \nu') Y_2(\boldsymbol{r}, \boldsymbol{C}', \nu')\,d\Omega$$

$$-\Big[(C_\theta^2 + C_\varphi^2)\,\frac{\partial Y_1}{\partial C_r} + (C_\varphi^2 \cot\theta - C_r C_\theta)\,\frac{\partial Y_1}{\partial C_\theta} - (C_\varphi C_\theta \cot\theta + C_r C_\varphi)\,\frac{\partial Y_1}{\partial C_\varphi}\Big] - C_\theta\,\frac{\partial Y_1}{\partial\theta}. \tag{1.3}$$

Equation (1.2) describes gas slip over a rigid flat surface, and Eq. (1.3) corrects the slip velocity to surface curvature.

We seek for the solution of Eqs. (1.2) and (1.3) in the form

$$Y_1(\boldsymbol{r}, \boldsymbol{C}, \nu) = C_\theta\varphi_1(x, C_r) + C_\theta(C_\theta^2 + C_\varphi^2 + \nu^2 - l - 1)\varphi_2(x, C_r)$$

$$+ \varphi_3(x, C_r) + \gamma(C^2 + \nu^2 - l - 1/2)\varphi_4(x, C_r); \tag{1.4}$$

$$Y_2(\boldsymbol{r}, \boldsymbol{C}, \nu) = C_\theta\psi_1(x, C_r) + C_\theta(\nu^2 - l + 1)\psi_2(x, C_r), \tag{1.5}$$

where $x = r - R$ and $\gamma^2 = 1/(l + 1/2)$ [6].

Expansion (1.4) extends the splitting of the steady-state BGK equation in problems of slip and temperature jump for a monatomic gas at the edge of a rigid flat surface [8] to the case of a molecular gas. The functions $\varphi_1(x, C_r)$ and $\varphi_2(x, C_r)$ describe the thermal and isothermal slip, and the functions $\varphi_3(x, C_r)$ and $\varphi_4(x, C_r)$ describe the temperature jump. Plots of $\varphi_3(x, C_r)$ and $\varphi_4(x, C_r)$ for molecular gases can be found in [6]. Expansion (1.5) allows explicit identification of the dependence of $Y_2(\boldsymbol{r}, \boldsymbol{C}, \nu)$ on ν and takes into account that the distribution function in slip problems is proportional to the mass-velocity component C_τ tangential to the surface. For a spherical surface, the coefficient C_τ coincides with C_θ.

We substitute Eqs. (1.4) and (1.5) into Eqs. (1.2) and (1.3) and take into account orthogonality (in the scalar-product sense) of polynomials in expansions (1.4) and (1.5); then, we obtain the following system of equations for the functions $\varphi_i(x, \mu)$ and $\psi_i(x, \mu)$ $(i = 1, 2)$:

$$\mu \frac{\partial \varphi_1}{\partial x} + \varphi_1(x, \mu) = \frac{1}{\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} \exp(-\tau^2) \varphi_1(x, \tau)\, d\tau; \tag{1.6}$$

$$\mu \frac{\partial \varphi_2}{\partial x} + \varphi_2(x, \mu) = 0; \tag{1.7}$$

$$\mu \frac{\partial \psi_1}{\partial x} + \psi_1(x, \mu) = \frac{1}{\sqrt{\pi}} \int\limits_{-\infty}^{+\infty} \exp(-\tau^2) \psi_1(x, \tau)\, d\tau + \mu \varphi_1(x, \mu) - 2 \frac{\partial \varphi_1}{\partial \mu}$$

$$+ 2\mu \varphi_2(x, \mu) - 4 \frac{\partial \varphi_2}{\partial \mu} - [\varphi_3(x, \mu) + \gamma(\mu^2 + 1/2)\varphi_4(x, \mu)]; \tag{1.8}$$

$$\mu \frac{\partial \psi_2}{\partial x} + \psi_2(x, \mu) = 4\mu \varphi_2(x, \mu) - 2 \frac{\partial \varphi_2}{\partial \mu}. \tag{1.9}$$

Here $\mu = C_r$ and orthogonality of the functions $f(\boldsymbol{r}, \boldsymbol{C}, \nu)$ and $g(\boldsymbol{r}, \boldsymbol{C}, \nu)$ implies that the integral $\int f(\boldsymbol{r}, \boldsymbol{C}, \nu) g(\boldsymbol{r}, \boldsymbol{C}, \nu)\, d\Omega$ equals zero.

We consider now the boundary conditions for Eqs. (1.6)–(1.9). Using the above-derived expression for $f(\boldsymbol{r}', \boldsymbol{v}, \omega)$ and relations (1.1), (1.4), and (1.5), we find the boundary conditions far from the particle surface:

$$\varphi_1(\infty, \mu) = \varphi_2(\infty, \mu) = \psi_1(\infty, \mu) = \psi_2(\infty, \mu) = 0. \tag{1.10}$$

To construct the boundary conditions on the particle surface, we first write out the distribution function over the gas volume. We seek for this function in the form

$$f^0(\boldsymbol{r}', \boldsymbol{v}, \omega) = f_0(\boldsymbol{r}', \boldsymbol{v}, \omega)[1 + \Psi(\boldsymbol{r}, \boldsymbol{C}, \nu)], \tag{1.11}$$

where $f_0(\boldsymbol{r}', \boldsymbol{v}, \omega)$ is the absolute Maxwellian with prescribed parameters on the particle surface.

In constructing $f^0(\boldsymbol{r}', \boldsymbol{v}, \omega)$, we take into account that, according to the asymptotic theory of rarefied gas flows near a solid surface at low Knudsen numbers [7], the mass velocity, the temperature, and the gas pressure involve two components: hydrodynamic and kinetic. The hydrodynamic components of the expansions satisfy the system of Stokes equations and have the order of the linear size of the wetted body (in the case of interest, the particle radius). The kinetic components play a noticeable role only in a thin gas layer immediately adjacent to the wetted surface, whose thickness is of order the mean free path of gas molecules (Knudsen layer); their characteristic scale is, therefore, commensurable with the mean free path of gas molecules.

In view of the aforesaid, to construct $f^0(\boldsymbol{r}', \boldsymbol{v}, \omega)$, we pass to a new scale in the configuration space. We redefine the dimensionless coordinate so that the dimensional radius-vector becomes $\boldsymbol{r}' = R'\boldsymbol{r}$ (again, we denote the new dimensionless coordinate as $\boldsymbol{r}$). Then, the equation for $\Psi(\boldsymbol{r}, \boldsymbol{C}, \nu)$ acquires the form

$$C_r \frac{\partial \Psi}{\partial r} + \frac{1}{r}\Big[C_\theta \frac{\partial \Psi}{\partial \theta} + \frac{C_\varphi}{\sin\varphi} \frac{\partial \Psi}{\partial \varphi} + (C_\theta^2 + C_\varphi^2) \frac{\partial \Psi}{\partial C_r} + (C_\varphi^2 \cot\theta - C_r C_\theta) \frac{\partial \Psi}{\partial C_\theta} - (C_\varphi C_\theta \cot\theta + C_r C_\varphi) \frac{\partial \Psi}{\partial C_\varphi}\Big]$$

$$= k^{-1}\Big[\pi^{-3/2} \int\!\!\int\!\!\int \exp(-\boldsymbol{C}'^2) K(\boldsymbol{C}, \boldsymbol{C}') \Psi(\boldsymbol{r}, \boldsymbol{C}')\, d\boldsymbol{C}' - \Psi(\boldsymbol{r}, \boldsymbol{C})\Big]. \tag{1.12}$$

As the ratio between the right and left sides of Eq. (1.12) is of the order of Kn^{-1}, the solution of this equation can be constructed using the method of successive approximations. We represent $\Psi(\boldsymbol{r},\boldsymbol{C})$ as a power series expansion in terms of k:

$$\Psi(\boldsymbol{r},\boldsymbol{C})=\psi^{(0)}(\boldsymbol{r},\boldsymbol{C})+k\psi^{(1)}(\boldsymbol{r},\boldsymbol{C})+k^2\psi^{(2)}(\boldsymbol{r},\boldsymbol{C})+\dots\,. \tag{1.13}$$

With allowance for Eq. (1.13), we expand the hydrodynamic components of the tangential mass velocity, temperature, and gas pressure in powers of k:

$$\begin{gathered}U_\theta=U_\theta^{(0)}+kU_\theta^{(1)}+k^2U_\theta^{(2)}+\dots,\\ T=T_0(1+\tau^{(0)}+k\tau^{(1)}+k^2\tau^{(2)}+\dots),\\ p=p_0(1+p^{(0)}+kp^{(1)}+k^2p^{(2)}+\dots).\end{gathered} \tag{1.14}$$

Here p_0 is the pressure in the undisturbed gas flow far from the particle surface.

We insert Eq. (1.13) into (1.12) and equate the coefficients at identical powers of k to obtain the following system of recurrent relations for $\psi^{(n)}(\boldsymbol{r},\boldsymbol{C})$ $(n=0,1,2)$:

$$\psi^{(0)}(\boldsymbol{r},\boldsymbol{C})=p^{(0)}+2C_rU_r^{(0)}+2C_\theta U_\theta^{(0)}+(C^2+\nu^2-l-1/2)\tau^{(0)}; \tag{1.15}$$

$$\begin{gathered}\psi^{(n)}(\boldsymbol{r},\boldsymbol{C})=p^{(n)}+2C_rU_r^{(n)}+2C_\theta U_\theta^{(n)}+(C^2+\nu^2-l-1/2)\tau^{(n)}-\boldsymbol{C}\nabla\psi^{(n-1)}(\boldsymbol{r},\boldsymbol{C})\\ -\frac{1}{r}\Big[(C_\theta^2+C_\varphi^2)\,\frac{\partial\psi^{(n-1)}}{\partial C_r}+(C_\varphi^2\cot\theta-C_rC_\theta)\,\frac{\partial\psi^{(n-1)}}{\partial C_\theta}-(C_\varphi C_\theta\cot\theta+C_rC_\varphi)\,\frac{\partial\psi^{(n-1)}}{\partial C_\varphi}\Big].\end{gathered} \tag{1.16}$$

Expressions (1.15) and (1.16) define the distribution function (1.11) over the gas volume in Barnett's approximation. With allowance for Eq. (1.11), the expression for $f(\boldsymbol{r}',\boldsymbol{v},\omega)$ turns into

$$f(\boldsymbol{r}',\boldsymbol{v},\omega)=f_0(\boldsymbol{r}',\boldsymbol{v},\omega)[1+\Psi(\boldsymbol{r},\boldsymbol{C},\nu)+Y(\boldsymbol{r},\boldsymbol{C},\nu)]. \tag{1.17}$$

From here, taking into account that $U_r^{(0)}\Big|_S=U_\theta^{(0)}\Big|_S=0$, we obtain the following equation in the case of diffuse reflection of gas molecules from the surface:

$$Y(\boldsymbol{r},\boldsymbol{C},\nu)\Big|_S=-\Psi(\boldsymbol{r},\boldsymbol{C},\nu)\Big|_S. \tag{1.18}$$

We insert expansions (1.1) and (1.13) into Eq. (1.18) and equate the coefficients at identical powers of k to obtain

$$Y_i(\boldsymbol{r},\boldsymbol{C},\nu)\Big|_S=-\psi^{(i)}(\boldsymbol{r},\boldsymbol{C},\nu)\Big|_S,\qquad i=1,2. \tag{1.19}$$

In view of Eqs. (1.4), (1.5), and (1.19), we write the boundary conditions at the particle surface in the following form $(\mu>0)$:

$$\varphi_1(0,\mu)=-2U_\theta^{(1)}+2\mu S_{r\theta}^{(0)}+\Big(\mu^2-\frac{1}{2}\Big)\frac{\partial\tau^{(0)}}{\partial\theta}; \tag{1.20}$$

$$\begin{gathered}\psi_1(0,\mu)=-2U_\theta^{(2)}-2\mu\Big(\mu^2-\frac{1}{2}\Big)\frac{\partial^2\tau^{(0)}}{\partial r\,\partial\theta},\\ \varphi_2(0,\mu)=\frac{\partial\tau^{(0)}}{\partial\theta},\qquad\psi_2(0,\mu)=0,\qquad S_{r\theta}^{(i)}=\frac{1}{r}\frac{\partial U_r^{(i)}}{\partial\theta}+\frac{\partial U_\theta^{(i)}}{\partial r}-\frac{U_\theta^{(i)}}{r}.\end{gathered} \tag{1.21}$$

As the sought components $U_\theta^{(i)}\Big|_S$ $(i=1,2)$ in the series expansion of mass velocity with respect to the parameter k enter only the boundary conditions (1.10) and (1.21), below we can restrict ourselves to solving Eqs. (1.6) and (1.9) with the boundary conditions (1.10), (1.20), and (1.21).

Thus, the calculation of the molecular-gas slip velocity over the surface of a spherical aerosol particle reduces to solving Eqs. (1.6) and (1.9) with the boundary conditions (1.10)–(1.21).

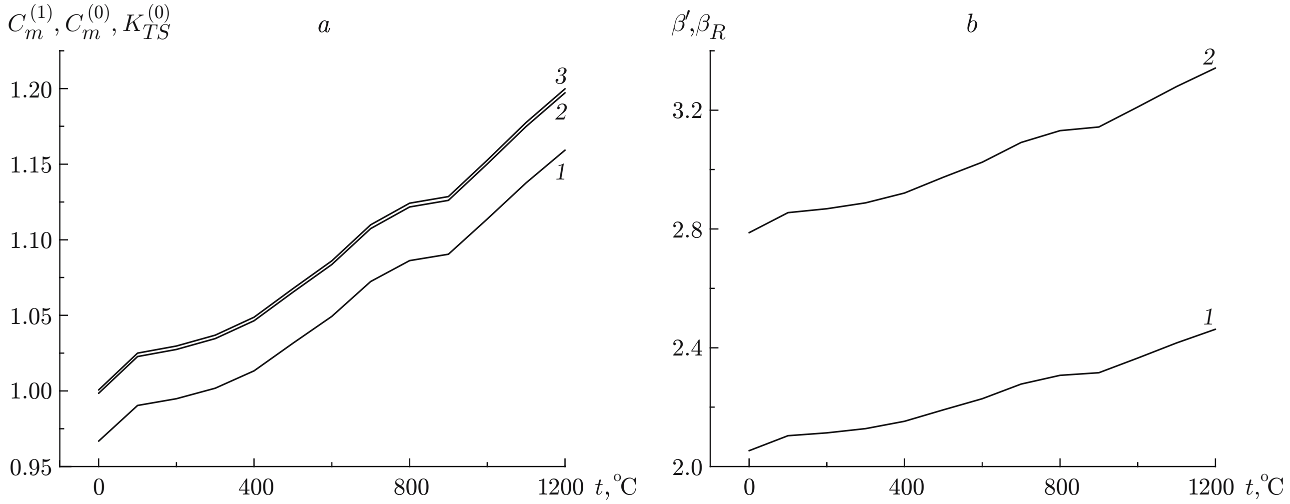

Fig. 1. Slip coefficients versus temperature for CO_2: (a) curves 1, 2, and 3 refer to $C_m^{(1)}$, $C_m^{(0)}$, and $K_{TS}^{(0)}$, respectively; (b) curves 1 and 2 refer to β' and β_R, respectively.

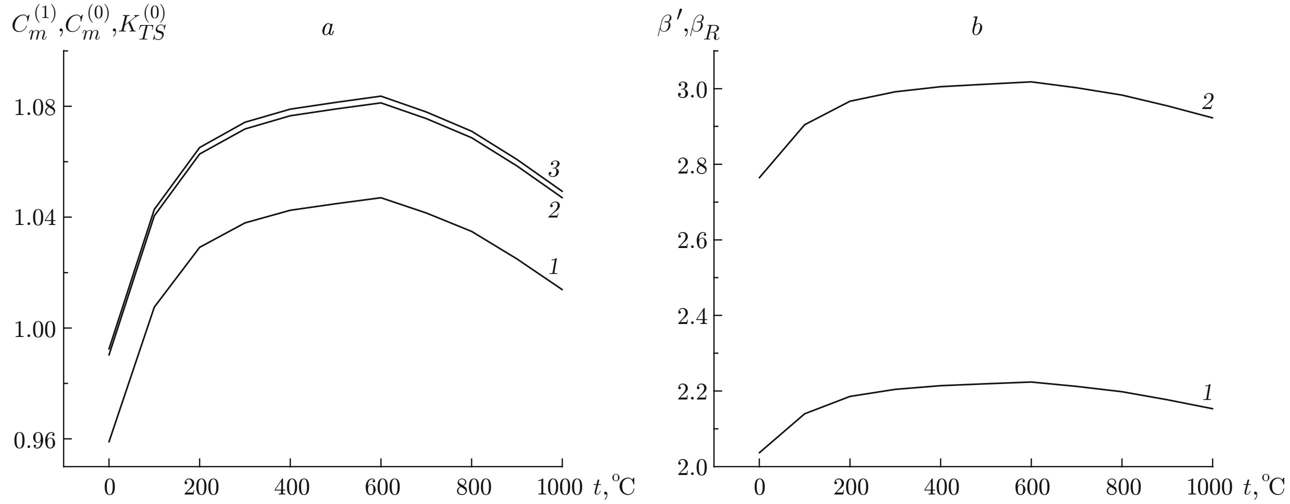

Fig. 2. Slip coefficients versus temperature for O_2: (a) curves 1, 2, and 3 refer to $C_m^{(1)}$, $C_m^{(0)}$, and $K_{TS}^{(0)}$, respectively; (b) curves 1 and 2 refer to β' and β_R, respectively.

2. Basic Results. System (1.6), (1.9) with the boundary conditions (1.10), (1.20), and (1.21) was solved using the method of elementary solutions (Case method) [8]. Taking into account Eqs. (1.14) and the results of [9–12], we write the rarefied-gas slip velocity over the spherical surface as

$$U_\theta\Big|_S = k\Big[k_1 S_{r\theta}^{(0)} + k_2\,\frac{\partial \tau^{(0)}}{\partial \theta}\Big] + k^2\Big[k_3 S_{r\theta}^{(0)} + k_4\,\frac{\partial \tau^{(0)}}{\partial \theta} + k_5\,\frac{\partial^2 \tau^{(0)}}{\partial r\,\partial \theta}\Big] + \ldots\ . \tag{2.1}$$

Here

$$k_1 = -Q_1 = 1.01619, \qquad k_2 = -(Q_2 - 1/2)/2 = 0.38316, \qquad k_3 = -1,$$

$$k_4 = Q_3 + Q_1 Q_2 = -0.53390, \qquad k_5 = [(Q_2 - 1/2)\varepsilon_T + Q_1 - 2Q_3 - \varepsilon_n]/2 + Q_3 - Q_1/2,$$

$Q_1 = -1.016\,19$, $Q_2 = -1.2663$, and $Q_3 = -1.8207$ are the Loyalka integrals [13]. For diatomic gases, $\varepsilon_T = 1.2168$, $\varepsilon_n = -0.6716$, and $k_5 = -0.7258$; for polyatomic gases, $\varepsilon_T = 1.1914$, $\varepsilon_n = -0.6525$, and $k_5 = -0.7388$.

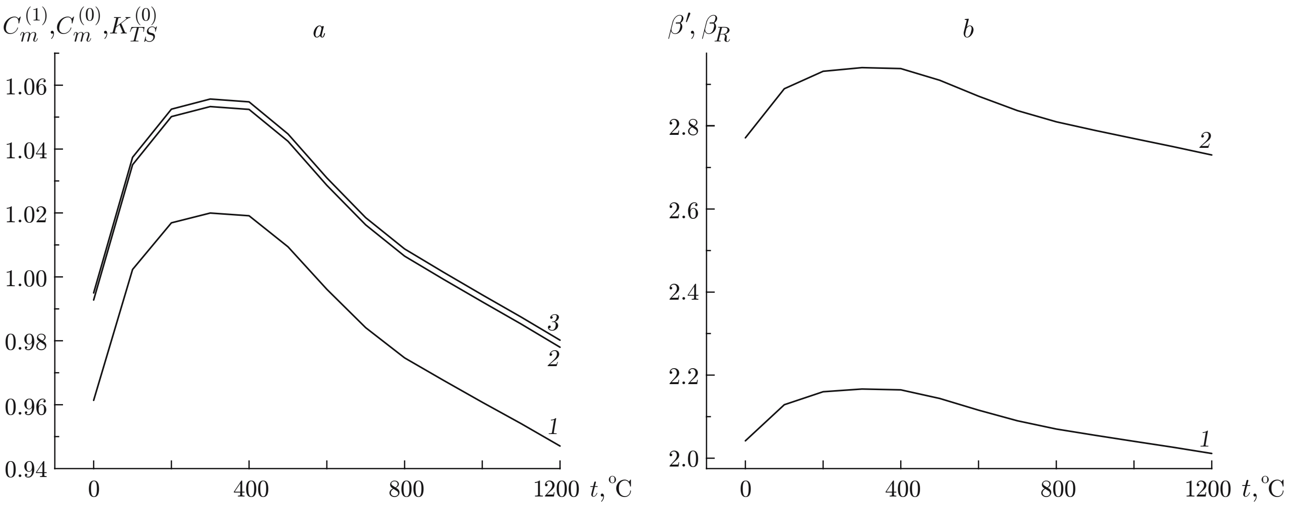

Fig. 3. Slip coefficients versus temperature for N_2: (a) curves 1, 2, and 3 refer to $C_m^{(1)}$, $C_m^{(0)}$, and $K_{TS}^{(0)}$, respectively; (b) curves 1 and 2 refer to β' and β_R, respectively.

Passing in Eq. (2.1) to dimensional variables and using the form commonly accepted in the kinetic theory of rarefied gases, we obtain

$$U'_\theta\Big|_S = C_m^{(0)}(1 - C_m^{(1)}\mathrm{Kn})\lambda \frac{\partial U'_\theta}{\partial r'}\Big|_\infty + K_{TS}^{(0)}(1 - \beta'\mathrm{Kn})\nu_{\mathrm{g}} \frac{\partial \ln T}{\partial \theta}\Big|_\infty - K_{TS}^{(0)}\beta_R\nu_{\mathrm{g}}\mathrm{Kn}\,\frac{\partial^2 \ln T}{\partial r'\,\partial\theta}. \qquad (2.2)$$

Here $C_m^{(0)} = 0.7645\mathrm{Pr}^{-1}$, $K_{TS}^{(0)} = 0.7662\mathrm{Pr}^{-1}$, $C_m^{(1)} = 0.7403\mathrm{Pr}^{-1}$, $\beta' = 1.5723\mathrm{Pr}^{-1}$, and $\beta_R = 2.1374\mathrm{Pr}^{-1}$ for diatomic gases and $\beta_R = 2.1757\mathrm{Pr}^{-1}$ for polyatomic gases.

Relation (2.2) defines the slip velocity of a molecular gas over a spherical surface with a small radius of curvature. It follows from Eq. (2.2) that allowance for rotational degrees of freedom of gas molecules makes the slip coefficients depend on the Prandtl number. As the Prandtl numbers for different gases vary in a rather wide range (for instance, Pr = 1.01 for water vapor at 100°C, 0.93 for ammonia, 0.85 for sulfur dioxide, and 0.64 for chlorine), taking into account such a dependence introduces substantial corrections to the gas slip velocity. There is not such a dependence for monatomic gases.

Allowance for rotational degrees of freedom of gas molecules makes the slip coefficients depend on gas temperature. The latter is caused by temperature-dependent thermophysical characteristics of the gas, such as its specific heat at constant pressure c_p, dynamic viscosity η, and thermal conductivity $\varkappa$. It is taken into account here that $\mathrm{Pr} = c_p\eta/\varkappa$ [14]. It is worth noting that the slip coefficients depend on gas temperature quite appreciably. For instance, the slip coefficient for carbon dioxide in the temperature range $t = 0$–1200°C vary by 16.6%, the slip coefficients for water vapor in the interval $t = 100$–700°C vary by 15.9%, and the slip coefficients of sulfur dioxide vary by 15.6% as the temperature changes from 0 to 1000°C.

The slip coefficients in (2.2) versus temperature are plotted in Figs. 1–3. The coefficients are seen to display a complicated behavior with varied temperature; as it follows from the expressions for slip coefficients derived above, this behavior is fully defined by the dependence of the Prandtl number of the gas on temperature.

Conclusions. The slip velocity of a molecular gas over the surface of a spherical aerosol particle with small radius of curvature is calculated is the present paper. It is shown that slip coefficients substantially depend on the Prandtl number and gas temperature.

The data obtained can be used, in particular, to predict the rate of sedimentation of aerosol particles in various filters and channels.

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